Dynamic Response of Moderately Thick Cylindrical Panels

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Theme

THIS paper considers the transient response of a circular cylindrical panel to arbitrary time-dependent loads. The study is based on the improved shell theory which includes effects of transverse shear and rotatory inertias. The governing equations of motion are solved by an explicit numerical procedure. Results are obtained for panels subjected to a time-varying normal load uniformly distributed over the entire panel surface. They are compared with findings based on the classical shell theory. Noteworthy differences between the two theories are obtained.

Content

The equations of motion of circular cylindrical shells based on the improved shell theory are obtained by specializing the general equations contained in Ref. 1 for cylindrical geometry. They can be written in the matrix form as follows:

$$AZ'' + BZ^{\cdot\cdot} + CZ'^{\cdot} + \tilde{D}Z' + \tilde{E}Z^{\cdot} + FZ = y + G\,\partial^2 Z/\partial^2 t \quad (1)$$

where t is the dimensionless time variable, $(\ )'$ and $(\ )^{\cdot}$ refer to differentiation with respect to the dimensionless space variables x and θ, respectively, and Z and y are defined as

$$Z = \begin{bmatrix} u \\ \beta_x \\ v \\ \beta_\theta \\ w \end{bmatrix}, \quad y = -k_1 \begin{bmatrix} q_x \\ 0 \\ q_\theta \\ 0 \\ q \end{bmatrix} \quad (2)$$

Here u, v, w are the components of the dimensionless displacement of a particle on the middle surface of the panel, β_x and β_θ the angles of rotation of the normals to the middle surface and q_x, q_θ, q the components of the dimensionless surface load. In the above, the matrices A, B, C, $\tilde{D}$, $\tilde{E}$, F and G and the constant k_1 which are expressed in terms of the material and geometric properties of the panel are given in Ref. 2.

The solution of Eq. (1) together with appropriate boundary and initial conditions is accomplished by an explicit numerical procedure. The derivatives with respect to the space variables, x and θ, and the time variable t are replaced by finite-difference representations. If the value of the vector Z is denoted by $Z_{i,j,p}$ where i and j refer to two space indices and p to a time index, the following relations are obtained:

$$Z_{i,j,p+1} = \varepsilon^2 G^{-1}(\bar{A}Z_{i+1,j,p} + \bar{B}Z_{i,j,p} + \bar{C}Z_{i-1,j,p} - \bar{Y}_{i,j,p}) \quad (3)$$

where

$$\bar{A} = A/\Delta_1{}^2 + \tilde{D}/2\Delta_1, \quad \bar{B} = -2A/\Delta_1{}^2 - 2B/\Delta_2{}^2 + F + 2G/\varepsilon^2,$$
$$\bar{C} = A/\Delta_1{}^2 - \tilde{D}/2\Delta_1 \quad (4)$$

$$\bar{Y}_{i,j,p} = y_{i,j,p} + (G/\varepsilon^2)Z_{i,j,p-1} - \bar{\bar{D}}Z_{i-1,j-1,p} - \bar{\bar{E}}Z_{i,j-1,p} +$$
$$\bar{D}Z_{i+1,j-1,p} + \bar{D}Z_{i-1,j+1,p} - \bar{F}Z_{i,j+1,p} - \bar{D}Z_{i+1,j+1,p} \quad (5)$$

and

$$\bar{D} = C/4\Delta_1\Delta_2, \quad \bar{E} = B/\Delta_2{}^2 - \tilde{E}/2\Delta_2, \quad \bar{F} = B/\Delta_2{}^2 + \tilde{E}/2\Delta_2$$

In Eqs. (3–5) Δ_1 and Δ_2 are, respectively, the dimensionless increments in the x and θ directions and ε is the dimensionless time increment. Examination of Eq. (3) shows that if Z has been determined at every spatial point i, j on two time lines p and $p-1$, one can directly calculate the value of Z for the next time increment $p+1$, thus advancing the solution in time. The solution values needed to start the whole process are obtained from initial conditions. Since the study of dynamic problems often requires determination of solutions at hundreds or even thousands of time steps, Eq. (3) is not best suited for direct numerical computation of $Z_{i,j,p+1}$ for the reason that time consuming matrix operations must be performed at each time step. Instead, all the matrix operations are carried out mathematically to yield a system of five algebraic equations which can be used for direct determination of the numerical values of the components of the vector $Z_{i,j,p+1}$. These equations are contained in Ref. 2 and are omitted here.

Since an explicit numerical procedure is described here for the step-by-step solution of Eq. (3) the time increment must be chosen small enough to insure numerical stability. The following relationships are employed to guide the proper selection of the magnitude of the time increment $\Delta\bar{t}$ for a given space mesh size $\Delta\bar{x}$.

$$\Delta\bar{t} \leqq \Delta\bar{x}\,(\rho(1-\nu^2)/E)^{1/2} \quad (6)$$

$$\Delta\bar{t} \leqq \frac{2\Delta\bar{x}}{(4(\bar{a}/\bar{b}) + (\bar{c}/2\bar{b})(\Delta\bar{x})^2 + \{[4(\bar{a}/\bar{b}) + (\bar{c}/2\bar{b})(\Delta\bar{x})^2]^2 - 64D/\bar{b}\}^{1/2})^{1/2}}$$

where

$$\bar{a} = \rho h^3/12 + D\rho/G', \quad \bar{b} = \rho^2 h^3/12G', \quad c = \rho h$$

and

$$G' = \pi^2 E/24(1+\nu) \quad (7)$$

In the above equations, $\bar{t}$ and $\bar{x}$ are the time and space variables. E, ν and ρ are Young's modulus, Poisson's ratio and density of the panel material, h is the thickness of the panel and $D = Eh^3/12(1-\nu^2)$. Equations (6) and (7) are derived, respectively, from the consideration of the in-plane wave propagation and the dynamic bending behavior of a flat plate.[3] It should be mentioned that the small value of $\Delta\bar{t}$ obtained from Eqs. (6) and (7) must be used as a first approximation and subsequent numerical experimentation can be used to determine its true critical value.

The numerical procedure described here is applied to determine the response of a circular cylindrical panel to a time-dependent lateral load uniformly distributed over its entire surface. The load has a time history in which there is a rapid, linear pressure rise over a short time interval followed by a relatively slow linear pressure decay over a long time interval. The influence of the boundary conditions on the dynamic behavior of the panel is studied by considering both clamped and simply supported edges. A separate computer program based on the classical shell equations of motion is written to provide results for comparisons with present findings.

The geometric and material properties of the panel considered are $a = 3$ in., $h = 0.3$ in. and 0.6 in., $l = 12$ in., $\theta_o = 120°$, $E = 10.5 \times 10^6$ psi, $\rho = 0.25 \times 10^{-3}$ lb-sec²/in.⁴ and $\nu = 1/3$. (a, h, l, θ_o are, respectively, radius, thickness, length and angle of opening of panel.) The normal loading on the panel has a maximum value of 10 psi. The accuracy of the numerical results is established in a pragmatic way. It involves a successive

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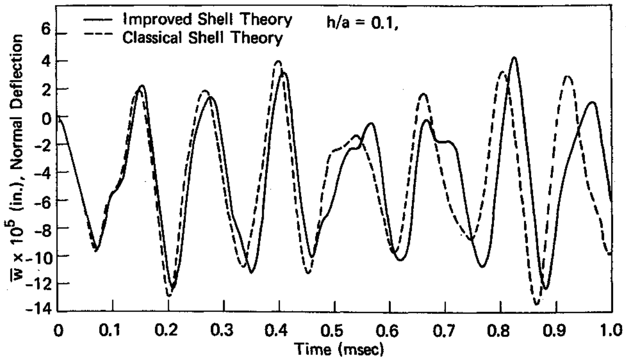

Fig. 1 Normal deflection at center of panel with clamped edges vs time.

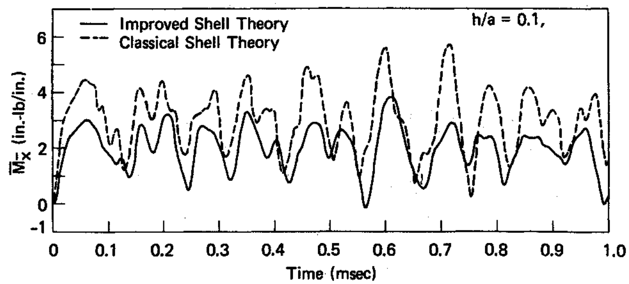

Fig. 2 Bending moment at center of edge of panel with clamped edges vs time.

decrease of the sizes of time and space increments until the results appear to converge to a limiting solution.

Figure 1 shows a comparison of the normal deflections at the center of the panel obtained from either the improved or the classical shell theory. It is seen that the magnitudes of the deflection predicted by the classical theory are slightly different from those derived from the improved theory. The extent of the differences is less for panels with clamped edges than that for panels with simply supported edges, and it increases with the increase of the panel thickness in the cases investigated. Furthermore, the deflections obtained from the classical theory oscillate at higher frequencies than those given by the improved theory. This is physically plausible since neglecting the effects of transverse shear and rotatory inertias tends to increase the rigidity of the shell.

Figure 2 is the plot of the bending moment at the edge of the panel. It can be seen that significant differences exist between the results from the two theories. The values of the largest amplitude of bending moments (occurring in the time interval considered) predicted by the classical shell theory are significantly higher than the corresponding values obtained from the improved shell theory. The differences vary approximately from 15% (at the center) to 50% (at the edge) in the examples studied. The large difference at the edge is expected due to the effect of the Kirchhoff boundary conditions employed in the classical theory. Such an effect decreases as the distance from the edge increases. The presence of large differences in the values of the bending moments derived from two theories was also found by Medige, Lin and Reismann.[4]

In conclusion, a numerical procedure is presented which can determine the dynamic response of circular cylindrical panels to arbitrary time-dependent loads. The effects of transverse shear and rotatory inertias on the transient behavior of the panel are included. Application is made to determine the transient response of cylindrical panels to a time-varying normal load uniformly distributed over the entire panel surface. Both deflections and bending moments are obtained and compared with findings based on the classical shell theory. It is found that the panel deflections predicted by classical shell theory are slightly different from those derived from the improved shell theory while the values of the maximum amplitudes of bending moments (occurring in the time interval considered) predicted by the classical shell theory are significantly higher than the corresponding values based on the improved shell theory. In view of the results of the present study, it is recommended that improved shell theory should be used to determine the dynamic behavior of moderately thick shells. The use of classical shell theory would overestimate the magnitudes of stresses in the shell.

References

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